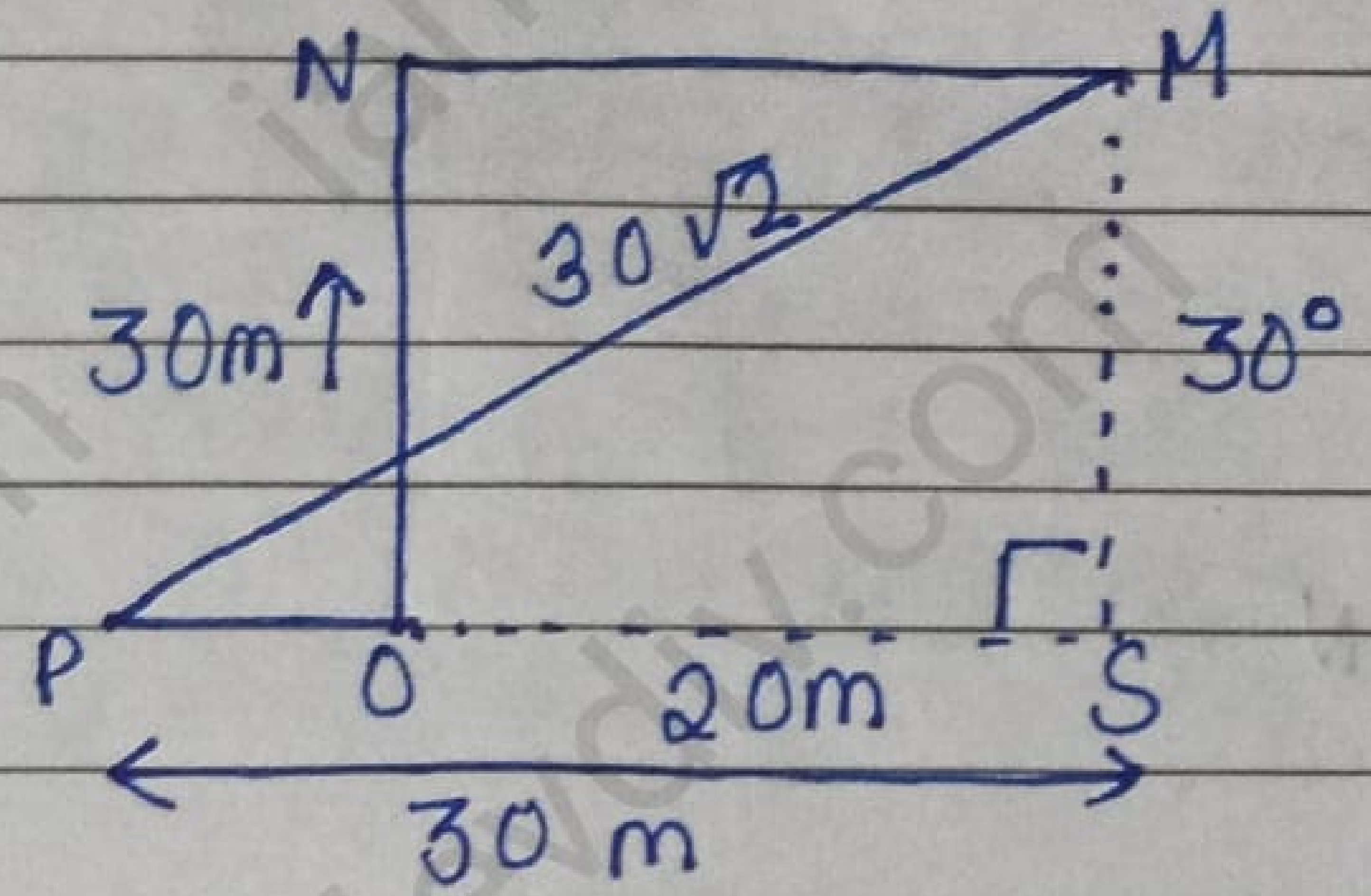
Ques - A person moves 30m north & then 20 m towards east & finally $30\sqrt{2}$ m in South west direction. The displacement of the person from origin will be.



By pythagoras theorem

$PM^2 = MS^2 + PS^2$

$(30\sqrt{2})^2 - (30)^2 = PS^2$

$1800 - 900 = PS^2$

$900 = PS^2$

$30 = PS$

$PO = 30 - 20 = 10m$

10 m along west

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$= (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - B_xA_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

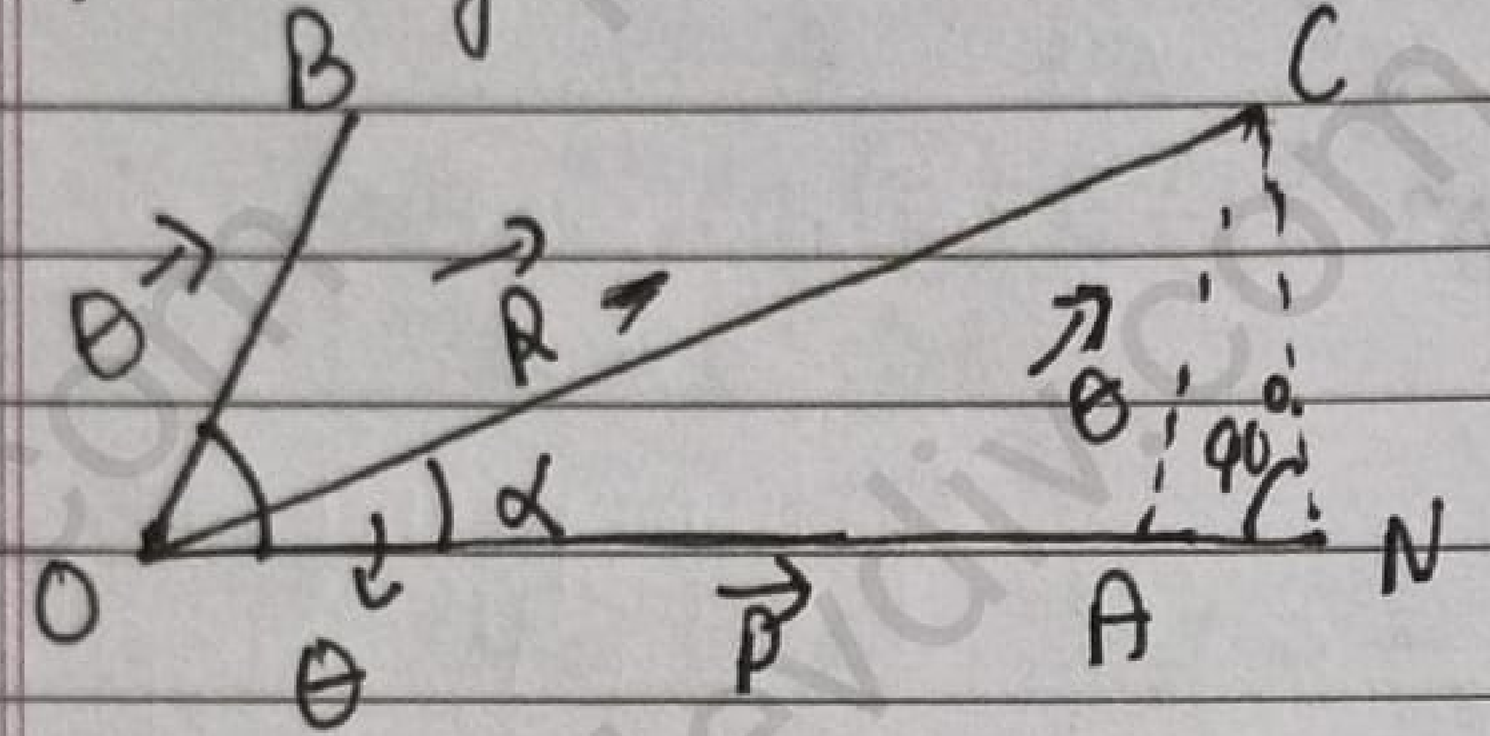
Q - $\vec{A} = 2\hat{i} + 4\hat{j} - \hat{k}$

$\vec{B} = \hat{i} - \hat{j} + 4\hat{k}$

$\vec{A}\times\vec{B} = ?$

$= (4\times4-(-1\times(-1))\,\hat{i} - (2(4)-1(-1))\,\hat{j} + (2(-1)-(4)(1))\,\hat{k}$
$= (16-1)\,\hat{i} - (8+1)\,\hat{j} + (-2-4)\,\hat{k}$
$= 15\hat{i} - 9\hat{j} - 6\hat{k}$

Parallelogram method



$\vec{OA} = \vec{P}$ $\qquad OC = \vec{R}$ (Assume)
$OB = \vec{Q}$ $\qquad \vec{R} = \vec{P} + \vec{Q}$
$\angle BOA = \theta$
θ b/w $\vec{P}$ & $\vec{Q}$
$\angle COA \neq \angle BOA$
$\angle COA = \alpha$

$OB \parallel CA$
$OB = \vec{Q}$ $\qquad CA = \vec{Q}$
$\angle BOA = \angle CAN = \theta$
$\angle CNA = 90^\circ$
$ON = OA + AN$
$ON = \vec{P} + AN$ — (1)

From ΔCNA
$\sin\theta = \dfrac{CN}{CA}$
$CN = CA\sin\theta$
$CN = \vec{Q}\sin\theta$ — (2)

Similarly
$AN = \vec{Q}\cos\theta$ — (3)

from ΔCNO (By PGT)

$OC^2 = CN^2 + ON^2$

$(\vec{R})^2 = CN^2 + (\vec{P} + AN)^2$

From values eq. 2 & 3

$R^2 = (\vec{Q} \sin\theta)^2 + (\vec{P} + \vec{Q}\cos\theta)^2$

$R^2 = Q^2 \sin^2\theta + P^2 + Q^2\cos^2\theta + 2PQ\cos\theta$

$= (Q^2\sin^2\theta + Q^2\cos^2\theta) + P^2 + 2PQ\cos\theta$

$= Q^2(\sin^2\theta + \cos^2\theta) + P^2 + 2PQ\cos\theta$

$= Q^2 + P^2 + 2PQ\cos\theta$

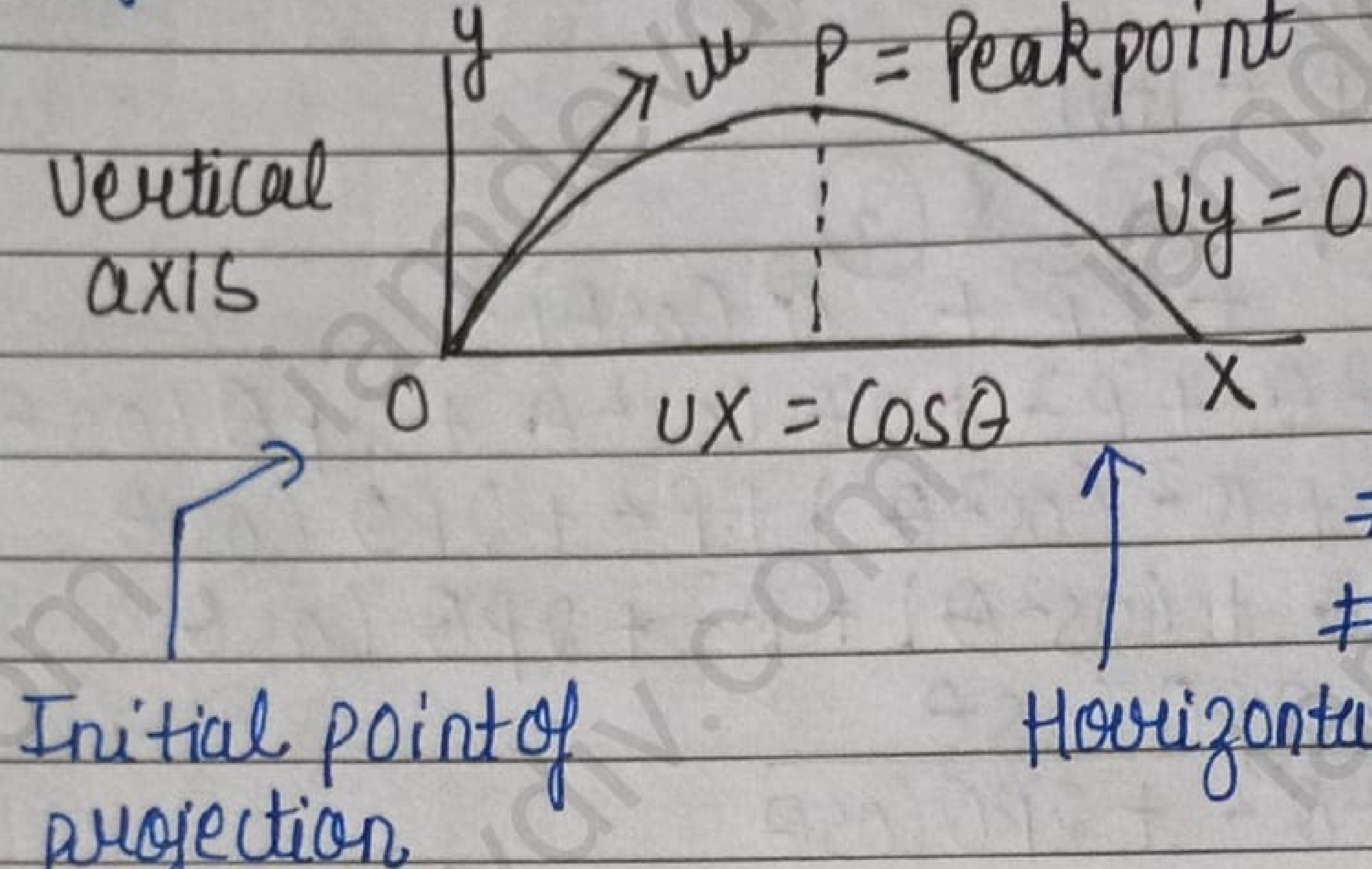
$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Projectile Motion

→ Projectile motion
→ Trajectory / Parabola path
→ Time of Ascent
→ Time of flight
→ Height of projection
→ Horizontal Range
→ Numericals
→ M.C.Q.'s
→ Case Study

Ques - Define the projectile motion. prove that a projectile particle or object always represent parabolic path during the motion. Reduce the derivation from time of Ascent, time of flight, Height of projection and Horizontal Range of a projected particle.

Ans - When an object thrown in vertical division at any angle $0° < \theta\ 90°$ along to the Horizontal axis under to the gravity. It follow a projectile motion/curved surface. This path is called projectile path.



u = initial velocity
# $0° < \theta < 90°$
# Under gravity

PM = Maximum height of projection
$UX = U\cos\theta$ = Horizontal component
$Vy = U\sin\theta$ = Vertical component

- $O \rightarrow P$

Initial to peak point → Time of Ascent

- OPN → Time of flight T

$T = 2t$

$$\text{Time of Ascent} = \frac{T\text{flight}}{2}$$

Motion along to x-axis

$U = UX = U\cos\theta$

$S = ON = x$

$a_x = 0$

from $S = Ut + \frac{1}{2}at^2$

$$X = UX + \frac{1}{2} \times 0 \times t^2$$

$$x = (U\cos\theta)t = t = \frac{x}{U\cos\theta} \quad \text{— (1)}$$

Motion into y-axis

$$U = U_y = U\sin\theta$$
$$S = y$$
$$a_y = -g$$
$$t = \frac{x}{u\cos\theta}$$
$$s = ut + \frac{1}{2} a_y t^2$$
$$y = \frac{\cancel{u}\sin\theta\, x}{\cancel{u}\cos\theta} + \frac{1}{2}(-g)\left(\frac{x^2}{u^2\cos^2\theta}\right)$$
$$y = (\tan\theta)\, x - \left(\frac{g}{2u^2\cos^2\theta}\right) x^2$$

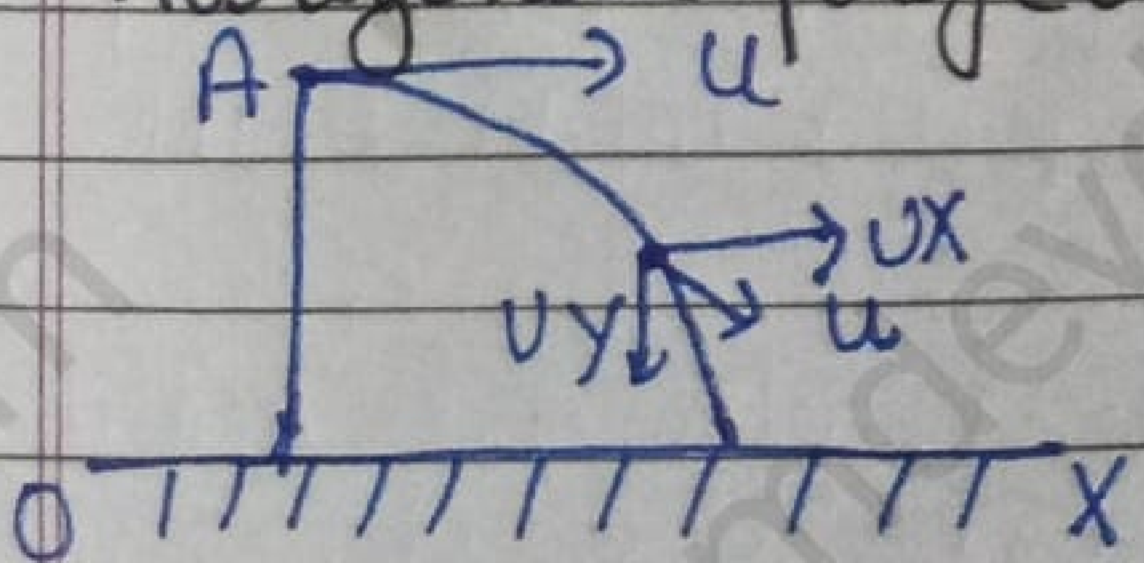
Time of Ascent

$$V = V_y = 0$$
$$U = U_y = U\sin\theta$$
$$a = -g$$
$$t = ?$$

$$V_y = U_y + a_y t$$
$$0 = u\sin\theta - gt$$

$$\boxed{t = \frac{U\sin\theta}{g}}$$

Horizontal projection



X - axis motion
$$X = ut$$
$$t = \frac{X}{u}$$

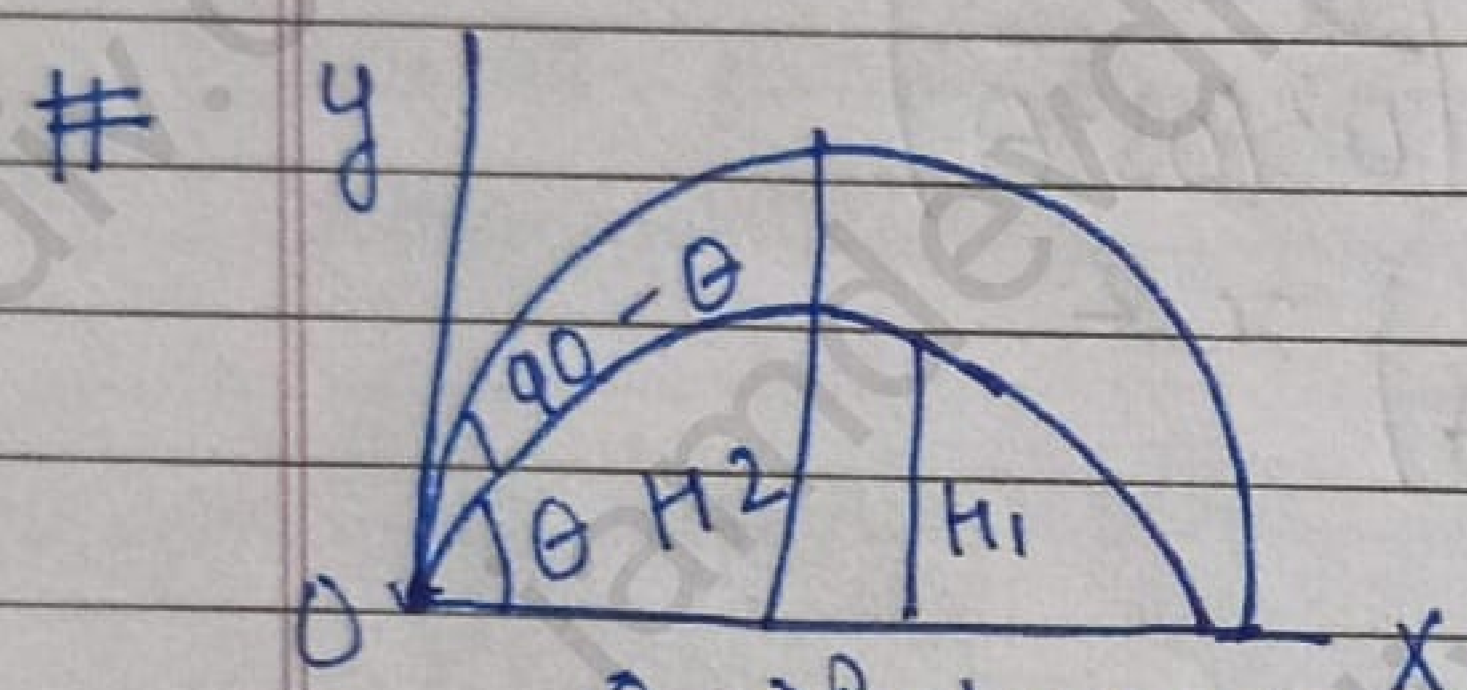
Y - axis motion

$y = ut + \frac{1}{2}at^2 \Rightarrow y = U_y t + \frac{1}{2}gt^2$

$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2$

$y = \frac{1}{2}\left(\frac{g}{u^2}\right)x^2 \Rightarrow y = ax^2$

$H = \frac{1}{2}gt^2 \Rightarrow \sqrt{\frac{2H}{g}} = T$



$\theta \rightarrow$ Body projection

$R_1 = R_2 \quad H_1 \neq H_2$

30 - 60°, 60 - 30

$R \propto \sin 2\theta$

Height of projetile motion

$S = H \qquad U = U_y$

$t = \frac{U\sin\theta}{g}$

from $S = Ut + \frac{1}{2}at^2$

$H = U\sin\theta \times \frac{U\sin\theta}{g} - \frac{1}{2}g\frac{U^2\sin^2\theta}{g^2}$

$H = \frac{U^2\sin^2\theta}{g} - \frac{1}{2}U^2\frac{\sin^2\theta}{g}$

$= \left(1 - \frac{1}{2}\right)\frac{U^2\sin^2\theta}{g}$

$H = \frac{U^2\sin^2\theta}{2g}$

If $\theta = 90°$ $\quad H_{MAX} = \frac{U^2\sin^2\theta}{g}$

Horizontal Range

$U \times T$

$= U\cos\theta \cancel{\frac{2U\sin\theta}{g}} \quad \frac{2U\sin\theta}{g}$

$= \frac{U^2}{g} 2\sin\theta\cos\theta$

$R = \frac{U^2 \sin 2\theta}{g}$

Ques Prove angles of projection for the same horizontal Range if there heights of projections are H_1 and H_2 Then find out a relation b/w R, H_1 and H_2

$R = 4\sqrt{H_1 H_2}$

$\theta_1 = \theta_2$

$\theta_2 = 90 - \theta$

$R = \frac{U^2}{g} \sin\theta\cos\theta$

$H_1 = \frac{U^2 \sin^2\theta}{2g}$

$H_2 = \frac{U^2 [\sin(90-\theta)]^2}{2g} = \frac{U^2\cos^2\theta}{2g}$

$H_1 H_2 = \frac{U^2 \sin^2\theta}{2g} \times \frac{U^2 \cos^2\theta}{2g}$

$$= \frac{1}{4}\left[\frac{u^2 \sin\theta \cos\theta}{2g}\right]$$

$$= \frac{1}{4}\left[\frac{R}{16}\right]^2$$

$$H_1 H_2 = \frac{R^2}{16}$$

$$R = 16 H_1 H_2 \rightarrow R = 4\sqrt{H_1 H_2}$$

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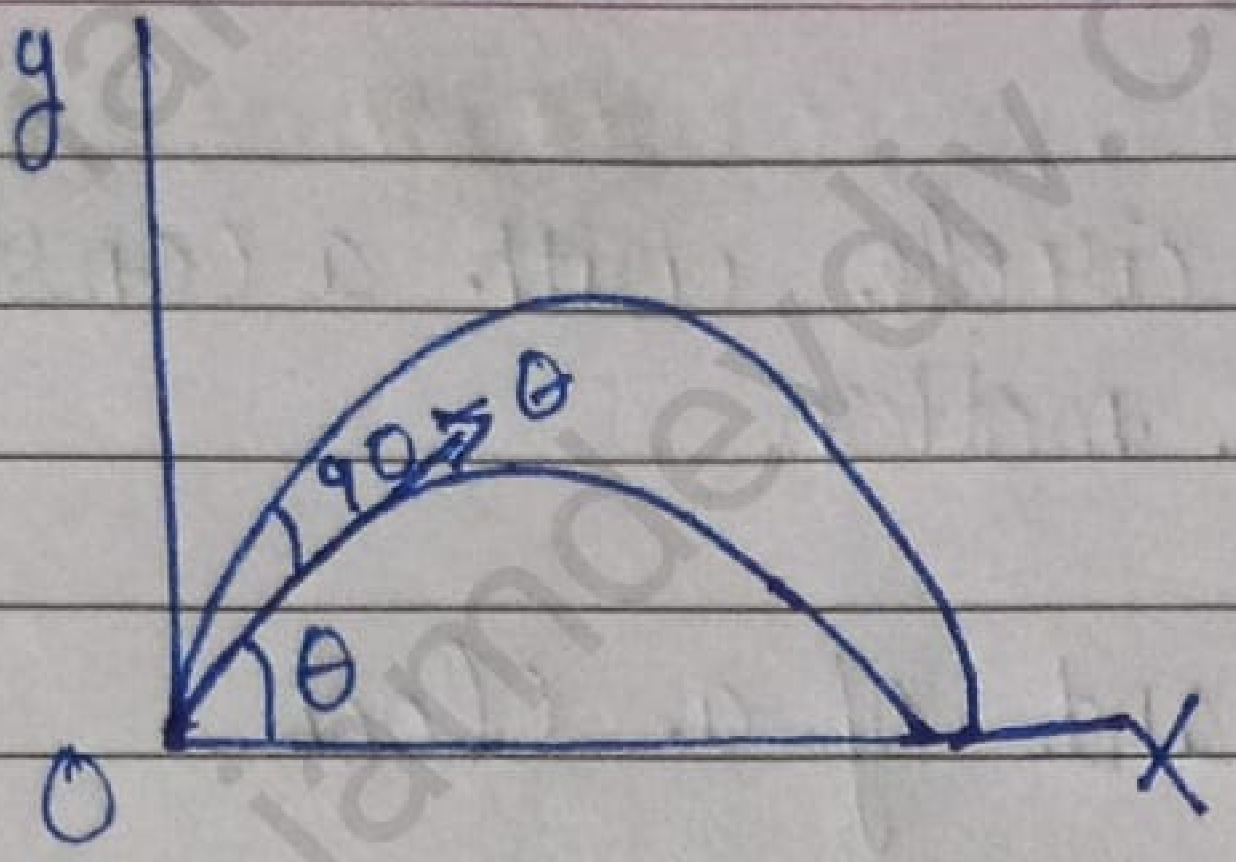


$T_1 \quad T_2$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

$$T = \frac{2u\sin\theta}{g}$$

$$\theta_1 = \theta \rightarrow T_1 = \frac{2u\sin\theta}{g} \quad -(1)$$

$$\theta_2 = 90-\theta \rightarrow T_2 \rightarrow \frac{2u\sin(90-\theta)}{g}$$

$$T_2 = \frac{2u\cos\theta}{g} \quad -(2)$$

(1) x (2)

$$T_1 T_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$$

$$= \frac{2}{g} \cdot \frac{u^2\, 2\sin\theta\cos\theta}{g} = \frac{2}{g} R$$

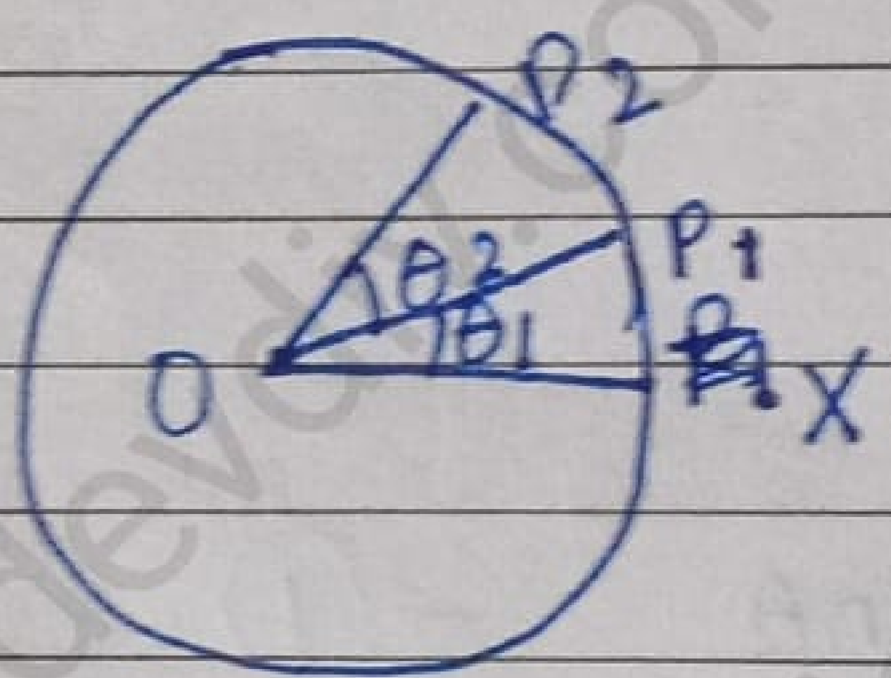
$$T_1 T_2 = \frac{2R}{g}$$

→ Circular motion:-
An object moving in a circle with a constant speed is called a circular motion

Ex:- Motion of the second hand of a clock.

→ Angular displacement:-

Change in angular position is called Angular displacement



$\theta_2 - \theta_1 = \Delta\theta$

$d\theta = \Delta\theta$

$P_1 P_2 = \text{Arc} = \Delta S$

$\frac{\Delta\theta}{\Delta t} \rightarrow \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$

$\frac{d\theta}{dt}$

$\theta = \frac{\text{Arc}}{\text{Radius}}$

$\theta = \frac{\Delta S}{r}$

→ **Angular velocity:-** The rate of change of Angular displacement is called Angular velocity.

Note:- $\omega = \frac{d}{dt}\theta = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t} = \frac{\text{Radian}}{\text{Second}}$ $[T^{-1}]$

(MCQ)
Note:- Relation b/w v, r & ω
v ↓ Velocity, r ↓ Radius, ω ↳ Angular velocity

Because we know that angular displacement $= \theta = \frac{\text{Arc}}{\text{Radius}}$

But Angular velocity $\omega = \frac{d\theta}{dt}$

$$\omega = \frac{d}{dt}(\theta) \quad - \text{②}$$

$$\omega = \frac{d}{dt}\left(\frac{\Delta s}{r}\right)$$

$$\omega = \frac{1}{r}\frac{ds}{dt}$$

$$r\omega = \frac{d}{dt}s$$

$$r\omega = v$$

$$\boxed{v = r\omega}$$

Relation between angular acceleration (α) Linear acceleration (a) and R

$$a = \frac{\Delta v}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$= \frac{d}{dt} v$$

$$\cdot \ \alpha = \frac{d}{dt} \omega$$

Because we know that

$$v = r\omega$$

differentiate both side w.r. to t

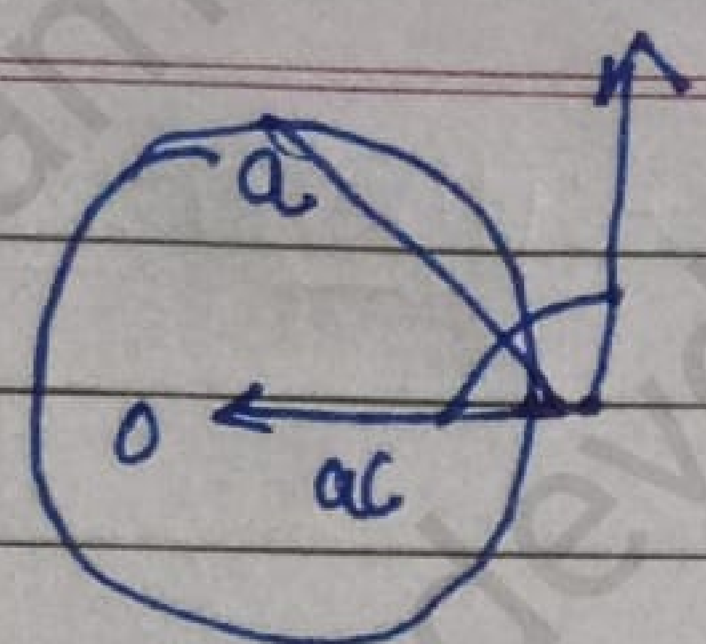
$$\frac{d}{dt} v = \frac{d}{dt} (r\omega)$$

$$a = r \frac{d}{dt} \omega$$

$$\boxed{a = r\alpha}$$

OR $\boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$

Note :-



$$\boxed{a = \sqrt{a_c^2 + a_T^2}}$$

$$F = \frac{mv^2}{r} = \frac{m(r\omega)^2}{r} = \frac{mr^2\omega^2}{r}$$

$$\boxed{F_c = mr\omega^2}$$

Centripetal acceleration :- Acceleration acting on a body moving in uniform circular motion is called centripetal acceleration. The direction of such type acceleration always radially towards the centre.

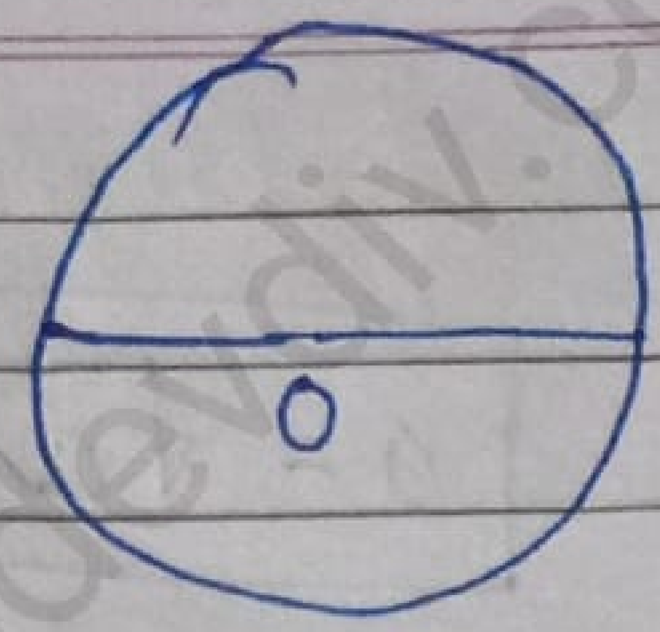
$$F = \frac{mv^2}{r}$$

$$\cancel{m}a = \frac{\cancel{m}v^2}{r}$$

$$\boxed{a_c = \frac{v^2}{r}}$$

$$a_c = \frac{(r\omega)^2}{r}$$

$$\boxed{a_c = r\omega^2}$$



$\frac{2\pi}{T}$

$\omega = 2\pi$